Christ Church Grammar School

2016
UNIT TEST 1

MATHEMATICS SPECIALIST Year 12

Section One:
Calculator-free

Student name _____

Teacher name _____

Time and marks available for this section

Reading time before commencing work: 2 minutes
Working time for this section: 15 minutes
Marks available: 15 marks

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. Write your answers in this Question/Answer Booklet.

2. Answer all questions.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that **you do not use pencil**, except in diagrams.

See next page

Question 1 (7 marks)

(a) Write $\frac{1+i\sqrt{3}}{1+i}$ in the form $x + yi$, where x and y are real numbers. (2 marks)

(b) By expressing both $1 + i\sqrt{3}$ and $1 + i$ in polar form $r \operatorname{cis} \theta$, show that $\frac{1+i\sqrt{3}}{1+i} = \sqrt{2}\left(\cos\left(\frac{\pi}{12}\right) + i \sin\left(\frac{\pi}{12}\right)\right)$ (3 marks)

(c) Hence, using your answers from parts (a) and (b), find the exact value of $\sin\left(\frac{\pi}{12}\right)$. (2 marks)

See next page

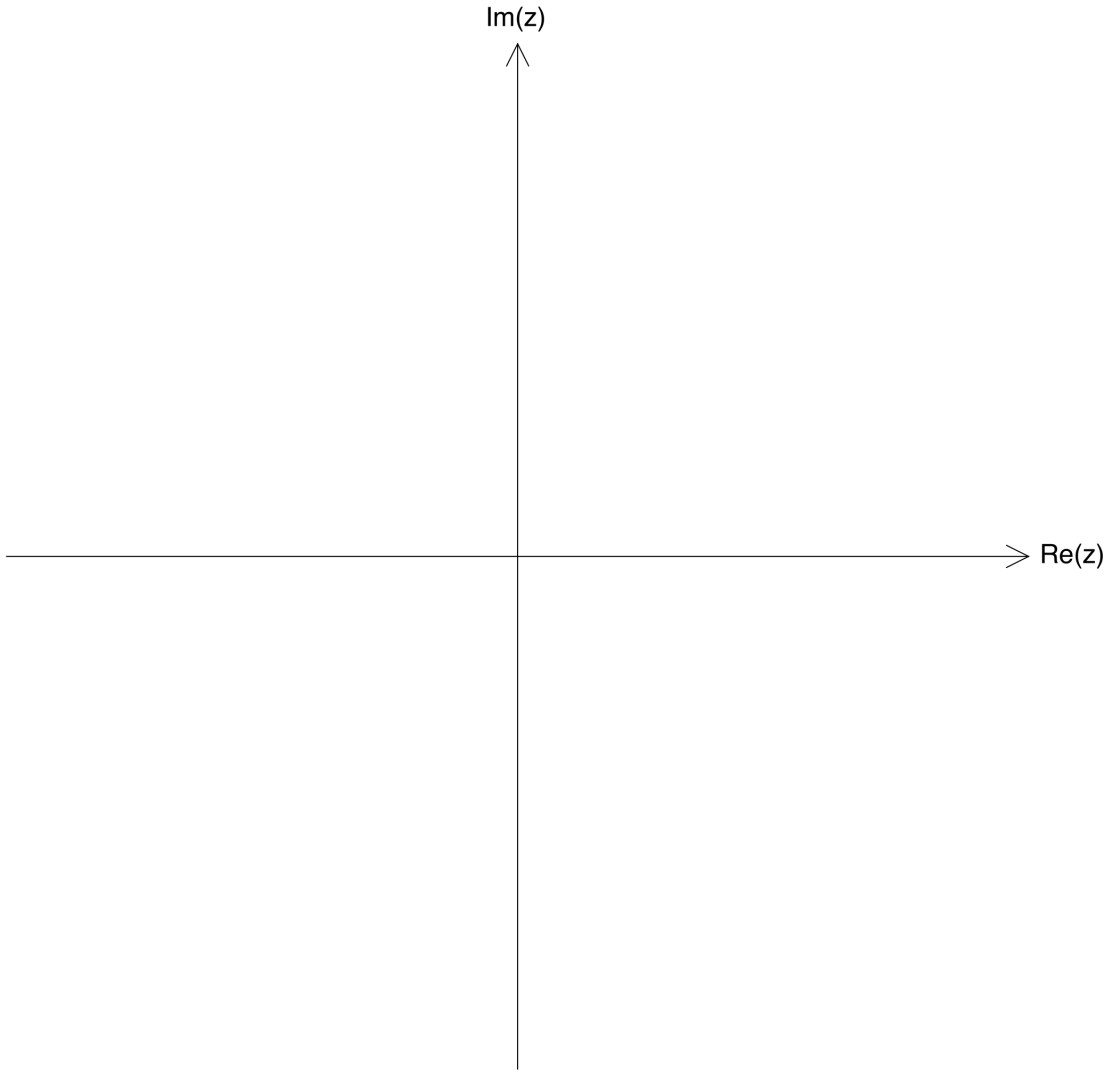
Question 2 (8 marks)

(a) Show that $(1 + i)^5 = -4 - 4i$. (3 marks)

(b) Hence determine all the roots of the equation $z^5 = -4 - 4i$, expressing each in the form $r \operatorname{cis} \theta$, with $r \geq 0$ and $-180° < \theta \leq 180°$. (3 marks)

See next page

(c) Sketch the roots from part (b) in the complex plane below. (2 marks)



End of questions

Christ Church Grammar School

2016
UNIT TEST 1

MATHEMATICS SPECIALIST Year 12

Section Two:
Calculator-assumed

Student name _____

Teacher name _____

Time and marks available for this section

Reading time before commencing work:	3 minutes
Working time for this section:	30 minutes
Marks available:	30 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. Write your answers in this Question/Answer Booklet.

2. Answer all questions.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that **you do not use pencil**, except in diagrams.

See next page

Question 3 **(7 marks)**

De Moivre's Theorem states that

$$(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\,sin(n\theta), \text{ for any integer } n.$$

(a) Prove de Moivre's Theorem for **positive** integers. (4 marks)

(b) Prove de Moivre's Theorem for **negative** integers. (3 marks)

See next page

Question 4 **(6 marks)**

(a) Expand $(\cos\theta + i\sin\theta)^5$ and write your answer in the form $a + ib$. (2 marks)

(b) Use de Moivre's Theorem and your result from part (a) to show that $\sin(5\theta) = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$. (4 marks)

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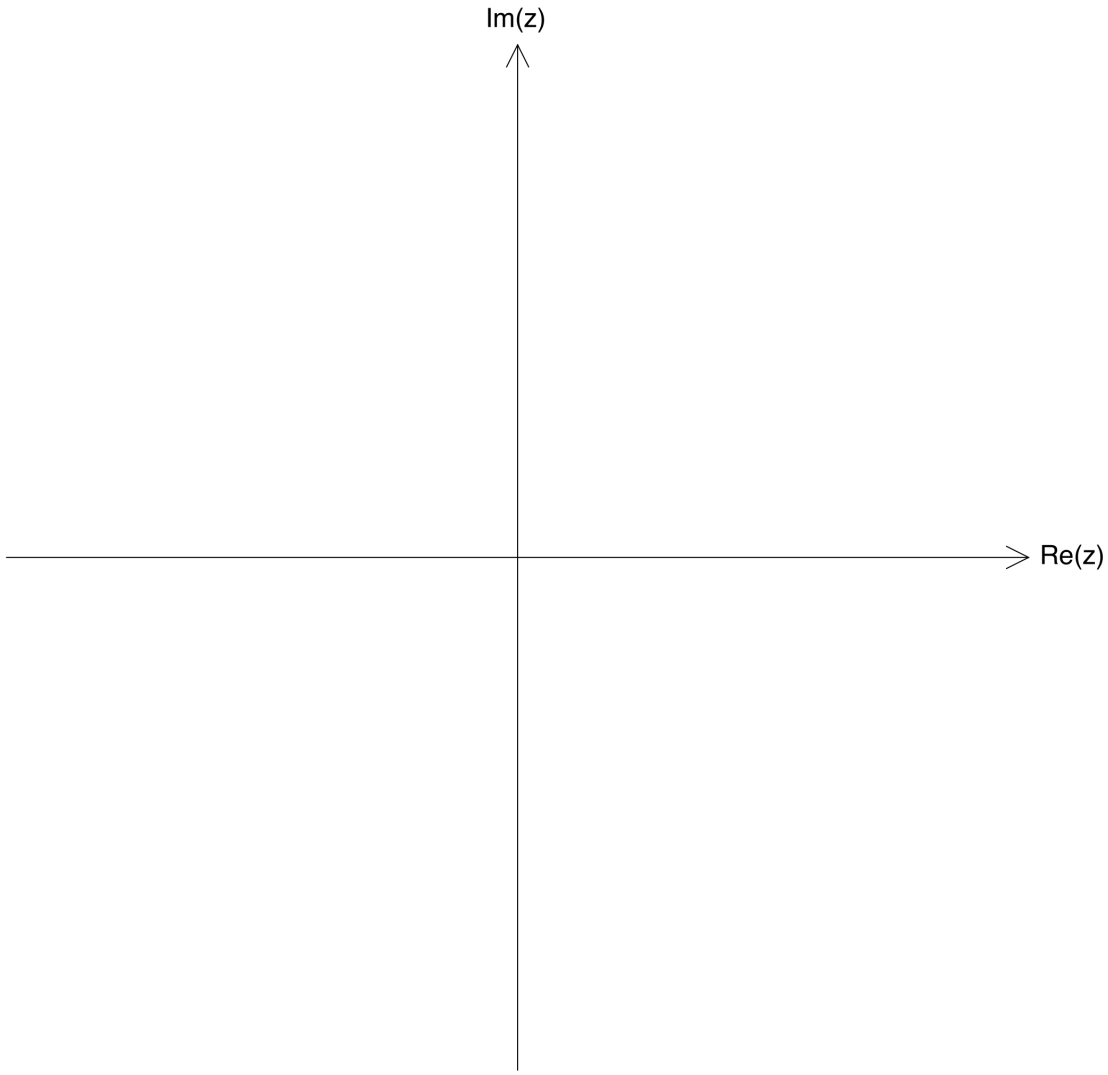
Question 5 **(5 marks)**

Consider the following sets of complex numbers:

$$S = \{z: |z + i| = |z - 2| \}$$

$$T = \left\{z: \left|\frac{z - 2i}{z + 2i}\right| = \sqrt{2} \right\}$$

Sketch the two sets of complex numbers in the Argand diagram below.

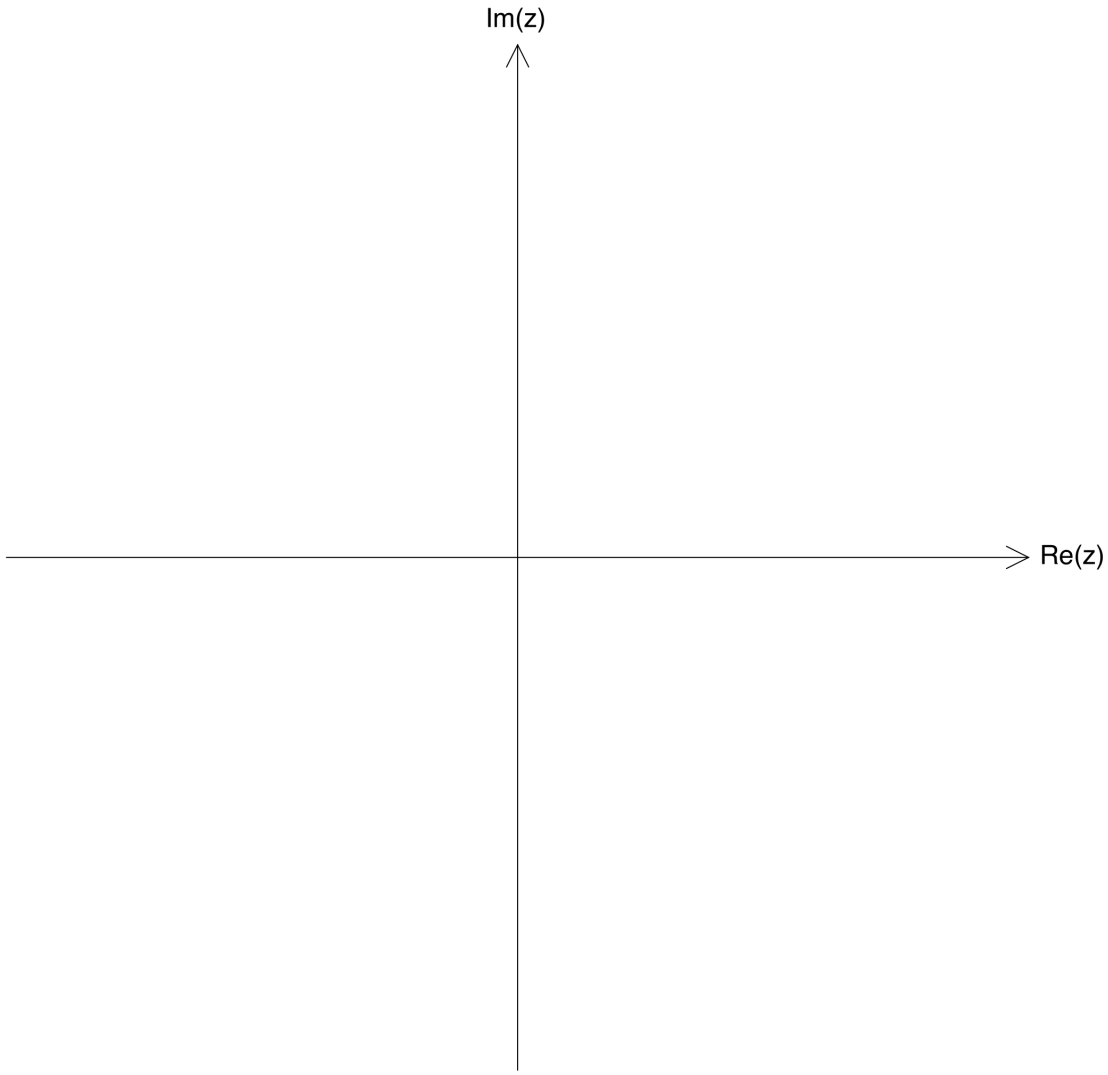


Question 6 **(7 marks)**

Consider the following set of complex numbers:

$$S = \{z: |z + 4 - 5i| = 3\}.$$

(a) Sketch the set of complex numbers S in the Argand diagram below. (2 marks)



(b) For z in S, determine the **maximum** value of $|z|$, the modulus of z. (2 marks)

(c) For z in S, determine the **minimum** value of $\arg(z)$, the argument of z, where $-\pi < \arg(z) \leq \pi$. (3 marks)

See next page

Question 7 **(5 marks)**

Show that, for every positive integer n, $(1+i)^n + (1-i)^n = 2\left(\sqrt{2}\right)^n \cos\left(\frac{n\pi}{4}\right)$.

End of questions